Dynamic logic.

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1. Introduction

This paper defines a dynamic variant of logic. The practical motivation is design of interactive information systems, e.g. interactive multimedia systems. Systems of this kind face the dilemma that they must be open to physical manipulation from the user (otherwise they were not interactive) but still must impose coherence and consistency upon their output (since otherwise they would merely produce fragmented scraps of information). On the one hand we want them to produce a structured discourse comparable to traditional types of discourses, on the other hand the user must be able to interfere with exactly this structure.

The solution explored in this paper is to stop viewing the narrative as a fixed linear structure — where interaction equals vandalism — and to begin to see the kernel of the story as a field of forces, some of which are created by the user, while others are made by the author. Another way of putting this is that the author should not see himself as the compiler of texts but as a creator of worlds, worlds which the reader can explore and use to generate texts.

I have treated some aspects of this problem domain elsewhere: Andersen & Holmqvist 1990 uses structural semiotics as a basis for designing interactive fiction, Andersen 1992, 1994 and 1995 discusses force fields in the context of catastrophe theory, and Andersen & Øhrstrøm 1994 suggests time logic as a component of a computer rhetoric.

Inspired by structural semiotics, this paper concentrates on the fundamental forces of narratives, as described by A. J. Greimas and others.

According to structural semiotics, the driving force of any discourse — its deep structure — is an opposition. Spy and detective novels exploit the opposition between Being and Seeming, Tarzan stories exploit the difference between Nature and Culture, and innumerable soap operas excel in Love versus Hate, Rich versus Poor, Good versus Evil. Negating the two opposites gives us the four terms that constitute Greimas' semiotic square, *the fundamental semantics* of a narrative.

	Truth		
	Being	Seeming	
Secret	Not Seeming	Not Being	Lie
	Falsity		

Fig. 1 The semiotic square.

The square in Fig. 1, borrowed from Greimas & Courtés 1979, has been exploited skilfully by John Le Carré, e.g. in “Tinker, Taylor, Soldier, Spy”. Our hero George Smiley *is* Loyal but does not *seem* so, since he has recently been sacked. Whereas Smiley belongs to the Secret axis, other characters occupy the Lie side; they *seem* Loyal but *turns out* to be double agents — like the new chief of the British Intelligence Agency who turns out to be a mole.

1.1. Narratives need a dynamic logic

The square, however, does not specify any stories, since it is a static *paradigm*. We need to convert it into linear *syntagms,* to unfold it in time, to create a *generative trajectory*, as Greimas calls it.

Smiley and his antagonist must be made to move around in the square. In the beginning of the book, the mole is chief of the British Intelligence, so he starts by Seeming (loyal) but not Being it. In the end he is exposed as a traitor, and changes to neither Being nor Seeming. Smiley, on the other hand, has just been fired when the book starts, but is approached and hired to make a discreet investigation. From the square of secrets, Being but not Seeming, Smiley succeeds in both Being and Seeming at last. His secret is disclosed, and he is made (temporary) chief of the spy organisation.

The operations moving subjects around among the predicates of the semiotic square are called *syntactic operations* by Greimas. They are quasi-logical of nature and essentially consist in negating or asserting predicates.

But since the task of the semiotic square is to explain dynamic phenomena, its static nature is somewhat awkward, and it seems worthwhile searching for concepts that are inherently dynamic and do not need logical rules imposed from the outside.

Petitot 1985 investigates catastrophe theory with a similar purpose in mind. Although the present paper stays closer to classical Boolean logic, it has the same objective as Petitot’s book: the idea is to conceive of concepts as *dynamic objects* that can simultaneously represent passive semantics and active syntax.

Informed by Greimas’ original proposal, we start by defining 4 components:

1. *A state-description.* It represents the current state of the actors. In the beginning, we would have: *Be(Smiley, Loyal), ¬Seem(Smiley, Loyal), ¬Be(Mole, Loyal), Seem(Mole, Loyal),*
2. *Consistency Rules,* corresponding to Greimas’ fundamental semantics. Fast “housekeeping” rules ensuring consistency at every stage. For example, if Carré’s cold war universe could be described as “nothing is

what it seems”, then Being and Seeming are antonyms, and at each stage a fast rule must ensure that Seeming automatically negates Being, and conversely: $\forall x, y{:}Seem(x, y) \rightarrow \neg Be(x, y)$.

3. *Narrative Rules*, corresponding to Greimas fundamental syntax. Slow rules that change the truth of the sentences and thereby set the story in motion. For example, the disclosure of the mole is effected by a negation converting *Seem(Mole, Loyal)* into *¬Seem(Mole, Loyal).* Narrative Rules operate on (1); they change the truth values of the concepts.
4. *Meta Rules*. Many narratives contain meta-rules operating on (2), the fundamental semantics. One example is mediation, also called synchretism in linguistics. Mediating stories often start with a pair of antonyms — Seeming versus Being, Culture versus Nature — and the point of the story is to construct a character who, alone of all, is able to span both predicates, in spite of their contrariety. Tarzan is such a figure, and possibly Smiley: in the end of the novel he is recognised as Loyal by all while really being so. But his outward recognition — his Seeming— is only temporary, and he will probably be replaced by a younger person. Meta Rules change the relationships between concepts; they operate on (2).

If we use dynamic objects, we can reduce these four levels to two. We need one level to take care of the dynamics and topology of the present state of the narrative and a second level that changes the first level. The former function is called *compensation*, the latter *perturbation*.

1. *Compensation*. Compensation corresponds to the Consistency Rules(2) and seeks to maintain consistency: if a narrative event places Smiley on top of Being, Seeming automatically kicks him off if he tries to straddle that concept too,
2. *Perturbation*. Perturbation upsets consistency and corresponds to Narrative Rules (3) and Meta Rules (4). For example, it could consist in a slow internal instability conflating the opposition between Being and Seeming (people “ought to” be what they seem, and seem what they are. The truth “will out” eventually).

The choice of the terms *compensation* and *perturbation* is motivated by the model suggested in Section 3.3. In the rest of the paper I give a formal description of compensation, suggest ways of specifying perturbations, and give examples of the interplay between perturbations and compensations.

1.2. Contradictions need a dynamic logic

Another problem with the quasi-logical conception is that it is unclear what happens in case of contradictions. Dilemmas, contradictions and mediation of contradictions are so basic to storytelling that we cannot ignore them as deviant cases, as we normally do in logic. In Carré's paranoid cold war atmosphere, loyalty to one power entails non-loyalty to other powers:

(1) $\forall x,y,z{:}\, Loyal(x,y) \wedge (z \neq y) \rightarrow \neg Loyal(x,z)$

The mole's dilemma is that, as a Marxist, he believes that in order to be loyal to England, he must be loyal to its working class, which again requires loyalty to the USSR. Hence in order to be loyal to his country he must be illoyal to it! I agree with Greimas that even if the fundamental syntax has logical nature, it cannot be a standard logic, since, whereas standard logic normally counts $\neg(A \wedge \neg A)$ among their axioms and uses reductio ad absurdum as a method of proof, Carré is quite happy with having $Loyal \wedge \neg Loyal$ in his book.

If we extend semiotic analysis to cover culture and daily life, the inadequate handling of dilemmas becomes even more problematic. The reason is that our daily world is full of dilemmas which we handle routinely and non-arbitrarily. For example, a common way of coping with dilemmas is to unfold them in time, so that no instant contains a contradiction, even if the truth-values of the problematic sentences change cyclically[1].

For example, parents should spend most of their energy on work if they want to be good workers, but if they want to qualify as good parents they must spend it on their children. If they want both, they must spend the major part of their time on work and on their children, and if these two activities do not contain overlapping actions, they are faced with a contradiction, $A \wedge \neg A$. Unfolding the contradiction in time simply means to sometimes neglect job and take care of children, sometimes the opposite. The truth value of A will neither be constant nor random, but oscillate regularly between true and false.

Apart from historically contingent dilemmas, like the work-children dilemma, there are also genuine logical dilemmas. Liar-paradoxes like "This sentence is false" have haunted logicians since ancient Greece. The problem is that it seems to oscillate between two truth-values, true and false, much in the manner of the haunted parents above. Herzberger (1982) and later Gupta, & Belnap (1993) gives a systematical account of these paradoxes in terms of stability and periodicity, and I shall try to apply their reasoning on the slightly more realistic examples below.

One example is the dilemma of critical philosophers, e.g. the Frankfurter school. They claim that all members of capitalist societies suffer from false consciousness, which must mean that the assertions made by those members are false or at least perverted. But critical philosophers are members of a capitalist society so they too must suffer from false consciousness. Hence the theory they have just asserted must be false or perverted; but if it is false that all members suffer from false consciousness, then at least one member of a capitalist society must exist that does not have false consciousness[2].

Radical constructivism has the same self-destructive tendency: if no necessary sentence exists, then that very statement must also be contingent, entailing that it is possible that necessary statements exist after all[3]!

In both cases we have three elements:

(1) A proposition *a* asserting a general feature of propositions.
(2) Self-application of *a* on *a* produces a new version of *a* contradicting the original version.
(3) But the contradiction opens a possibility of keeping *a* intact and maintaining the universe defined by *a*.

The interesting thing is not (1) and (2), that these world views are inherently inconsistent, but rather (3), the way they manage to overcome the inconsistency and maintain themselves, namely to spot and occupy the vacant exception opened by the inherent contradiction (the critical philosopher escapes miraculously from the ideological distortion of capitalism, the radical constructivist occupies the only position where necessary truth is possible).

1.3. Construction of a dynamic logic[4]

Dynamic logics can be built in many ways. In the remaining part of the paper I shall sketch a simple version that incorporates the four kinds of rules from Section 1.1 in one framework, and which is able to cope with contradictions. The construction is intended to be close to ordinary Boolean logic, but — unfortunately — seems to deviate on some points.

The construction runs as follows: imagine a space in which signifiers are scattered. For illustrative purposes, I shall only use a plane or a line as spaces, but realistic spaces contain many dimensions. The axes of the space represent properties, and points on the axes a value of that property (on property spaces, see e.g. Gärdenfors 1992 and Lotman 1990). Each signifier has a location in the space, i.e. either an (x,y) coordinate pair, or simply an x location on a line. The location of a subject means that the subject has the property values corresponding to the location. The location of a predicate represents its extension. For example, if two predicates are situated far from each other it means that no object has both properties at the same time, whereas this is possible if they are close. If a subject is located near a predicate it means that the sentence Subject + Predicate is true. The space and its predicates correspond to the *State Description* (1) in Section 1.1.

This space— plane or line — is modulated by *force fields,* each of which influences one or more subjects. The set of subjects influenced by a force field is called the *domain* of the field. The total field is built recursively by assigning simple fields to the predicates, and then modulate the simple fields according to the logical laws that are assumed to hold in the world.

Although — at least to my knowledge — logic systems have not previously been defined as force fields, social, psychic, and biological systems have all been analysed from this point of view. Examples from the 19th Century is Marxist political economy and Freudian psychology. Modern examples include Bourdieu’s analysis of cultural systems (Bourdieu 1993), Arnheim’s analysis of art (Arnheim 1974) and the work on neurophysiology in Pellionisz & Llinas 1982, 1983.

As my basic building block I use the potential defined by $y = x^2$. The exact formula is an empirical matter and the x^2 is only chosen for its simplicity. The subjects influenced by the potential simply slide along the surface of the curve until they find a resting place where the forces working on the subject are zero. For example, Fig. 2 represents a state of affairs where the sentence “The girl is young” is considered true, because the only resting place for “girl” in the space is above the predicate “young”.

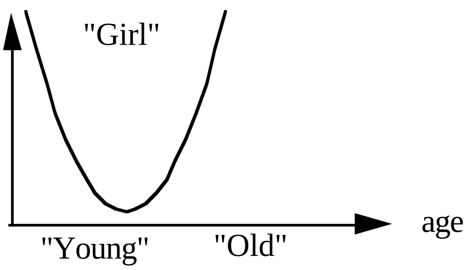

Fig. 2. Stability. “The girl is young”

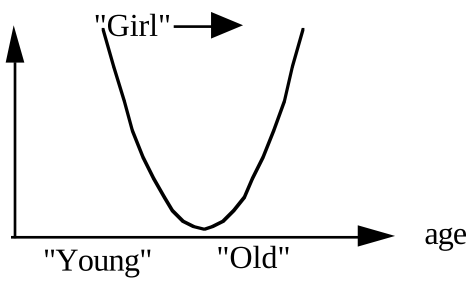

Fig. 3. Unstability. “The girl grows old”

The forces pushing subjects towards equilibria — if any exists — correspond to the fast *Consistency Rules (2)* in Section 1.1. The slow *Narrative Rules (3)* are represented by changes of the force field. For example, moving the potential to the right along the age dimension represents an ageing of the girl. As shown in Fig. 3, this perturbation will make “Girl” unstable and cause it to slide to the new equilibrium, above “old”.

In the following, I assume that there is no friction on the surfaces, and that the subject is only influenced by their gradient. On the other hand, I assume that the subject may possess a certain inertia, so that very small forces may not move it. The critical limit for movement is denoted ε. The limit is needed because we sometimes want the subject to rest, even if it is in fact influenced by very small forces.

2. Logic

This section presents one way of defining the force fields in the previous section. I present a full description of the *Consistency Rules* from Section 1.1, and discuss perturbations (*Narrative and Meta Rules*) informally. In the following I shall denote the force associated to an arbitrary expression E by f(E). There is a bit of mathematics involved in the following, but most of it has been put into footnotes in order not to bother readers with math fobia.

2.1. The logical connectives

Logical *or*, *P v Q*, is represented by a function consisting of the smallest values of f(P) and f(Q)[5].

Fig. 4 shows two predicates located in the same space but placed at different locations. The result of always taking the smallest value of the two curves, *P v Q,* is shown in fig. 5. Visual inspection shows clearly that the dynamic meaning of *P v Q* is that the subject can rest in P, it can rest in Q, or in both if their topology permits.

Fig. 4. Two concepts, P and Q.

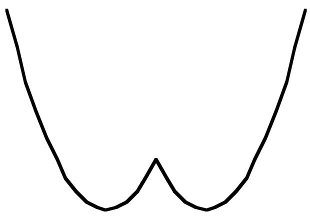

Fig. 5. The disjunction, $P \vee Q$

Fig. 5 represents the general law that subjects must either be P or Q or both, but cannot avoid one of them. For example, persons must either be young or old or both, but cannot escape one of the predicates.

We can now exemplify the last process mentioned in Section 1.1, namely the *Meta Rules (4)* changing the *Consistency Rules (2)*. Meta Rules do not move objects, but change the force fields associated to the predicates. In the dynamic setting, these rules are represented by changes of the energy landscape in Fig. 5.

An example is shown in Fig. 6, where we vary the distance between the two concepts. In the foreground of the drawing, P and Q have the same location, and we have only one minimum, so the subject can rest in both P and Q. As Q is displaced, a repulsor grows between them, marking the area covered by neither P nor Q. If $P \vee Q$ is to be true, the subject may not rest in this area, and this is indeed the case in our topology: neither the slopes nor the top of the ridge are equilibrium locations, so the subject will slide down into one of the valleys.

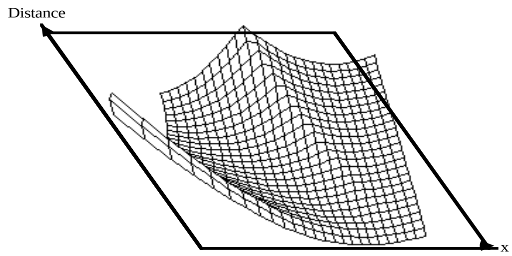

Fig. 6. Two predicates moving apart.

The bifurcation we experience on our tour from the foreground into the background of the picture represents a real change of reality. If P = "good worker" and Q = "good parent", then the bifurcation represents the increased difficulty of performing both duties satisfactory in modern society[6].

Logical *and* is represented by the maximum values of the component functions[7]. Look again at Fig. 4. The $\wedge$ operator is produced by always taking the highest value of the two curves. If they are close together, we get Fig. 7. Here the gradients at the bottom are so small that it works as a stable minimum representing the intersection of the two predicates.

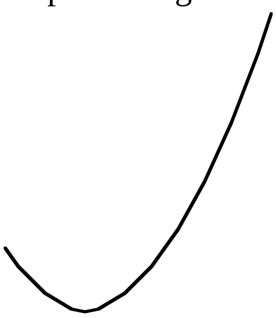

Fig. 7. $P \wedge Q$, P and Q close. There is an area of equilibrium in the bottom.

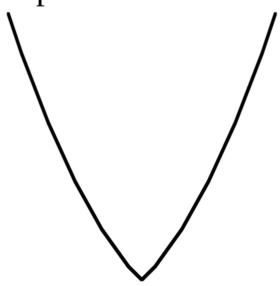

Fig. 8. $P \wedge Q$, P and Q distant. No equilibrium.

However, if we displace the predicates, the intersection disappears, the bottom gets pointed, with gradients so large that it no longer functions as a stable location (Fig. 8). In this situation the two predicates have no common space where the subject can rest. Placing a subject in the bottom will make it slide up and down the slopes, never reaching equilibrium (remember that there is no friction and that the subject is only influenced by the gradient of the surfaces which never becomes zero).

In the example with parents and workers the conjunction reads: “I am a good worker and a good parent”. If the two concepts are disjunct in reality, it is not possible to satisfy both requirements at the same time, and our subjects do what many parents do: in one week they try to be good parents, but have a bad conscience towards their workplace (“I am a good worker” is a lie), the next week the situation is reversed (“I am a good parent” is a lie).

This is the general method of representing dilemmas: situations where no equilibria exists and the subject must move for ever without rest. The pattern of oscillation occurs in every case where we try to achieve goals that are incompatible. For example, governments want to keep inflation and unemployment low, but according to the Philips curve these two magnitudes are inversely related,

$$inflation \approx \frac{1}{unemployment}$$

If it is true that low unemployment causes high inflation and vice versa, then the concepts *inflation* and *unimployment* are extensionally disjunct. The oscillation, caused by wanting *both* low inflation *and* low unimployment, shows up as the well-known stop-go policy.

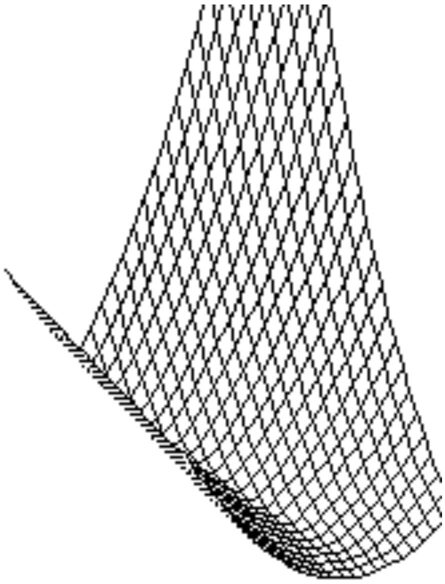

Fig. 9. Foreground of figure: P and Q overlap. Background: P and Q are disjunct

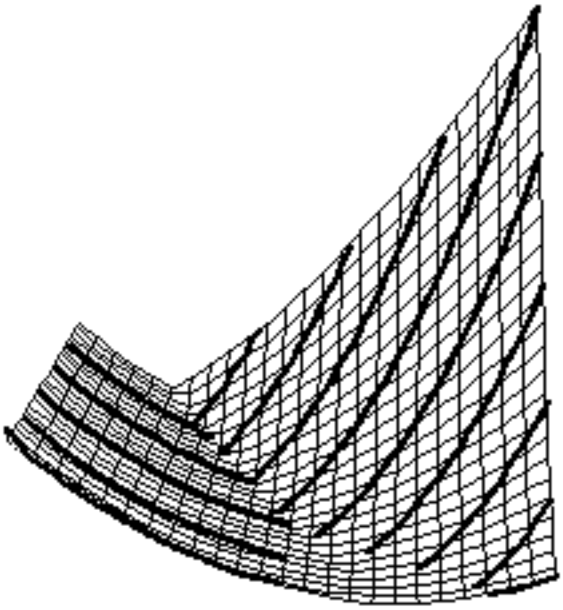

Fig. 10. As Fig. 9, with the flowlines shown. In the foreground bottom there is a section with no flowlines.

Fig. 9-10 shows this in 3D. When P and Q are sufficiently close together (foreground), the gradient approaches 0, and the subject can rest there. As P and Q depart, their overlap grows smaller, and finally, when they are disjunct, there is no stable place for the tired parent to rest. In this case —the background of Fig. 10— there is no way of making P $\wedge$ Q true.

If we travel from the background into the foreground, we experience that the oscillation between the two poles gradually diminishes and finally stops. This process is called *mediation* or *synchretism* and can be found in myths and popular stories that narrate how irreconcilable opposites approach and finally offer a resting ground for the hero. Stories of this kind need the 3D representation in Fig. 9 in order to represent both the ordinary actants and the mediator. In the Tarzan stories, only Tarzan can live in the foreground where equilibria exist between culture and nature: the animals, Negroes and white villains all live in the unstable background where each must chose either to belong to civilisation or to the jungle.

How should we represent negation? Suppose that a person A believes that the earth is a flat plane and is told it is not a plane but a ball. In this case, A suffers a major conceptual disaster, the subject "earth" quickly flees from the predicate "flat" and travels to "round". The example shows that a negation must turn an attractor into a repulsor, that is: it must change the sign of the gradients.

In addition, the strength of the new vector must be inversely related to the old one. If the gradient of "earth" in A's conception was near 0 when placed on "flat", the new information must turn the near-zero gradient into a large one that can effect the major conceptual reorganisation. If A had not really believed that the earth was flat, and therefore had placed "earth" well away from "flat" — for example over "ellipsoid" — the new information should not affect him fundamentally, since a minor adaptation suffices. Therefore negation should turn a large gradient into a small one[8].

Fig. 11 shows one potential that satisfies these criteria. The original predicate, P, is black whereas its negation, ¬P, is grey. Note that the slope of the negation has the opposite sign of the original, and that the absolute value of its gradient increases as the original's decreases, and conversely.

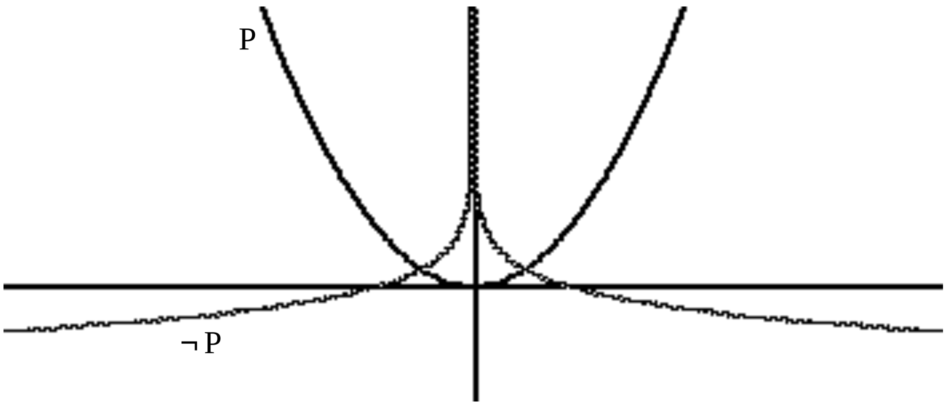

Fig. 11. A predicate (black) and its negation (grey).

By means of negation we can now represent antonyms like "old" and "young". Antonyms obey the rule that the same subject cannot rest on both predicates at the same time, viz. that $\neg(P \wedge Q) = (\neg P \vee \neg Q)$ must be true. ¬P and ¬Q are shown in Fig. 12.

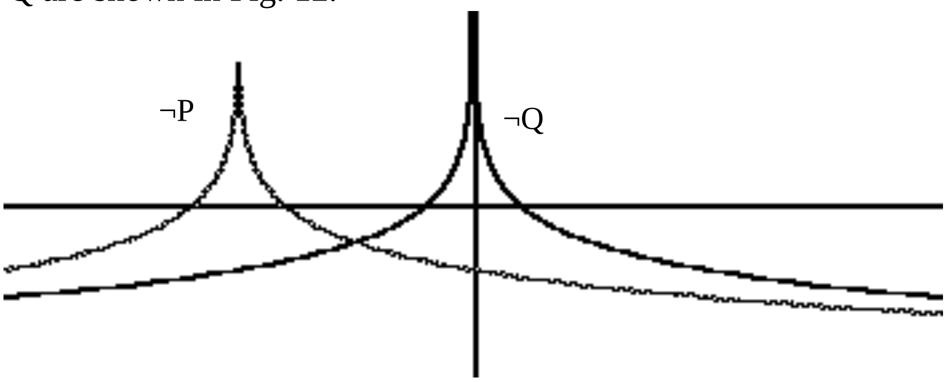

Fig. 12. ¬P, ¬Q

We see that both antonyms act as repulsors, and since we are concerned with an *or*-operation, it is the lowest values of both curves that influence stability. This gives us Fig. 13 as a representation of two antonyms.

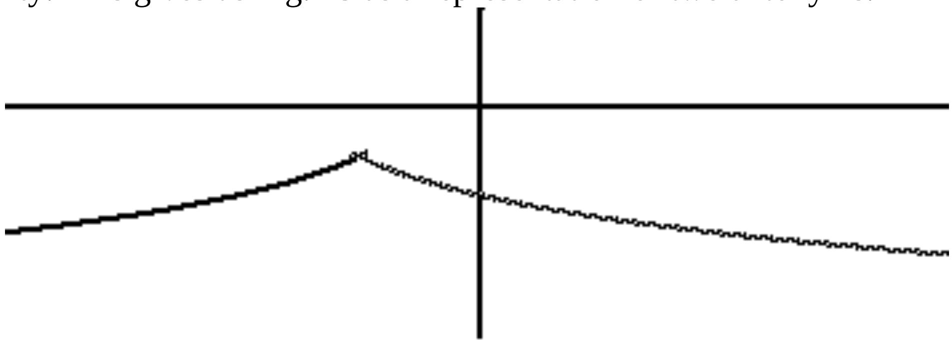

Fig. 13. ¬P ∨ ¬Q

The ridge between them prevents the subject from resting on both predicates at the same time; however, to the left and to the right in the diagram resting places will occur when the gradient becomes sufficiently small, allowing the subject to be either P or Q.

Fig. 14 shows what happens when we vary the distance between the two predicates: in the foreground they cover each other and this results in a large forbidden area. In the back they are far from each other, with only a very small unstable area as the result.

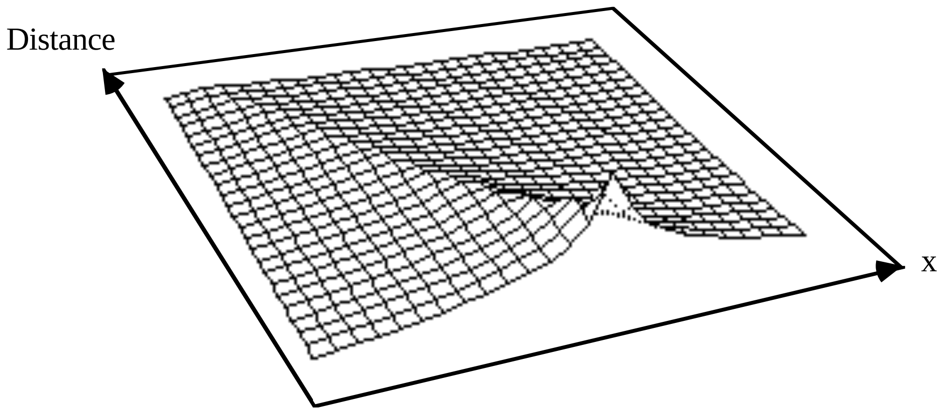

Fig. 14. Antonyms moving apart.

A journey from the back to the front of the picture could for example signify the real approach of phenomena that society wants to keep apart. An example could be racial segregation. In the background the two races lives in different locations, so at this time no one notices that Black and White are really conceived as antonyms. But then one of them is transported into the country of the other, either voluntarily or by force. As the two races approach in reality, the antonymness of Black and White becomes visible (the ridge in the foreground), and a racist ideology surfaces and take measures to enforce the segregation more and more explicitly.

The *logical* modulation of the conceptual landscape existed all the time, but because of the *factual* location of the concepts, it was only virtual. Nobody could see it. As the layout of the space changes, the modulation manifests itself clearer and clearer. Good examples of this process can be seen in recent years in almost any European country that receives refugees from the middle East and Africa.

2.2. Inference

Let us now turn to logical implication, $P \rightarrow Q$, which is the basis of well-known deduction rules like modus ponens and tollendo ponens:

$P \rightarrow Q$	$P \vee Q$
P	$\neg P$
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Consequent: Q	Consequent: Q

Since it is easiest to illustrate tollendo ponens, I shall only discuss this type of deduction. Fig. 15 displays the first antecedent, and Fig. 16 shows what happens when we add the second one. We see that the previous stable area over P has got a spike in it that will effectively expel any subject that seeks rest there. The only remaining resting place is over Q, which is exactly what the deduction says.

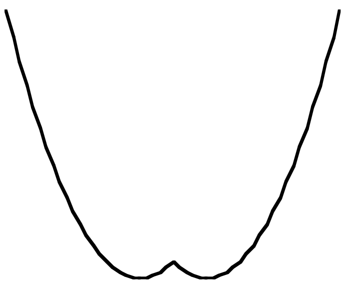

Fig. 15. $P \vee Q$

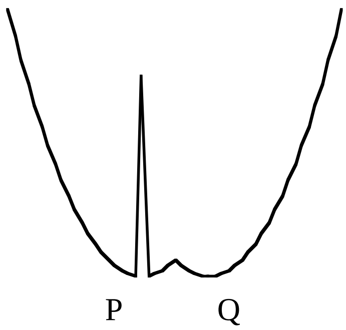

Fig. 16. $(P \vee Q) \wedge \neg P$

Finally, let us look at contradiction and tautology.

$P \wedge \neg P$ creates Fig. 17, which has no stable places — no subject can find peace here, as expected.

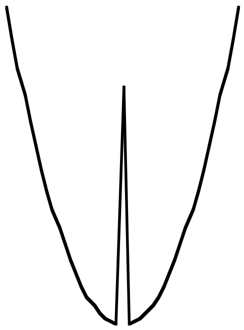

Fig. 17. Contradiction. $P \wedge \neg P$

Fig. 18. Tautology. $P \vee \neg P$

However, $P \vee \neg P$ does not give the desired result — zero gradient everywhere — since there is a crater over P where the gradient might be too strong for rest (Fig. 18). I don't know what to make out of it[9].

We have now seen that it is possible to design topologies and changes of topologies that closely resembles an ordinary Boolean algebra. However, there are some welcome differences.

2.3. Non-standard phenomena

We have already seen that the simple act of adding time to logic generates new phenomena that have a clearly human touch: child families will easily recognise the dynamics of Fig. 10. In this section we shall give a few more examples of this type.

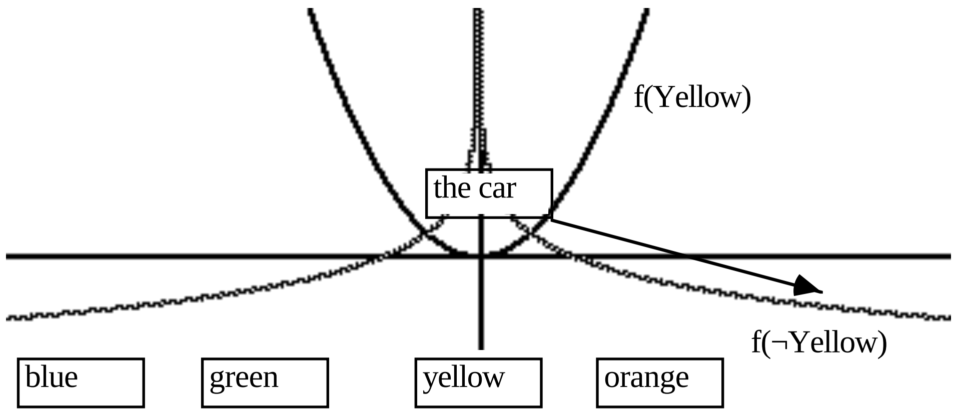

Fig. 19. "The car is yellow", "The car is not yellow"

According to the conventions we use, if we negate a predicate (Fig. 19), changing e.g. "The car is yellow" to "The car is not yellow", the subject will slide down the newly created ridge, and rest at the nearest available stability, giving us for example "The car is orange", "The car is green", etc.

In formal logic, we cannot immediately infer anything from a negation in isolation, but in our kind of dynamic topological logic, negation is a process taking place in a space with a basic metric of distance, which causes the subject to find a resting place “near” the original place. This seems to be paralleled in humans: if we hear “She is not blonde”, we may guess that “She is dark”, but not that “She is sick” or “She is director of the IBM”. We seem to stay within the same section of our space.

Finally, we will not have just truth and falsehoods, but different varieties of these concepts.

First we can distinguish between *cyclic* and *non-cyclic* truth-values. In the former type, the sentence shifts systematically between true and false, whereas in the latter it keeps one of the values.

Cyclic values were described in Section 1.2 and 2.1. Our example was P = good parent, Q = good worker, where P and Q are in fact disjunct. In this case the formula $P \wedge Q$ creates a ravine that is not differentiable at the bottom, so the subject will keep oscillating down there, never reaching a place with a sufficiently small gradient.

Non-cyclic values can be *stable* or *unstable*. The truth is *stable* when the subject rests at the bottom of a minimum, and *unstable* if it comes to rest on top of a maximum or saddlepoint (where the gradient is 0 too). Stable truth values stay true under large perturbations, whereas unstable ones change under small perturbations. Again the notion of stability of truths seem easily interpretable in humans. “The earth is round” stays true under large perturbations, whereas “I am a nice guy” may be endangered by very small insults (“I have never tried to cheat anyone!”).

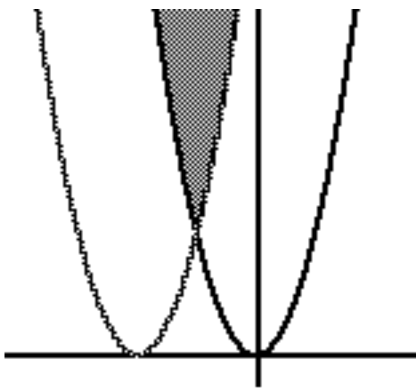

Fig. 20. $P \wedge Q$: the shaded area represents $P \wedge Q$.

The analysis of paradoxes as cyclic truth-values seems to fit human behaviour rather well: we do not rest immobile contemplating the paradox, but oscillate, trying at one time to satisfy one demand, at another time the other contradictory demand, or we try to change reality. In the next Section we shall look at a slightly more elaborated example of the latter possibility.

3. The "Schopenhauer" system

I start by describing the logic of a Romantic world view which I call "Schopenhauer" (the quotes means that the world view does not necessarily reflect the opinion of the real philosopher):

(2) $(\neg\text{life} \vee \neg\text{death}) \wedge (\neg\text{peace} \vee \neg\text{suffering}) \wedge (\text{life} \rightarrow \text{suffering}) \wedge (\text{peace} \rightarrow \text{death})$

Life and Death, Peace and Suffering are antonyms, so it is not possible for the romantic subject to rest on both of them at the same time. In addition, Life implies Suffering, and Peace implies Death.

3.1. Investigating the dynamics

In order to investigate the system, I built a small computer simulation where the predicates are placed in a two-dimensional plane. As usual, each predicate has associated a potential of the form x^2, with origin in the middle of the predicate, so without the logic there would be four basins, one in each corner. The logical formula modulates this original potential according to "Schopenhauer".

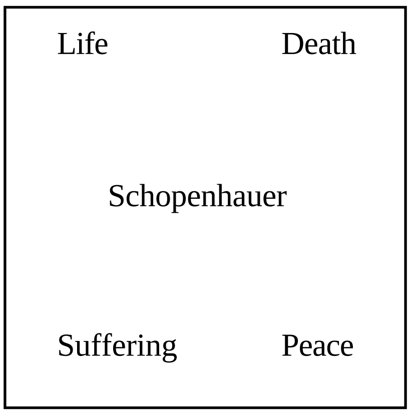

Fig. 21. The static version.

If we experiment with this logic, it turns out that the only stable place for a Subject is either on the Suffering or Death predicate or completely outside all the predicates — a conclusion the real Schopenhauer in fact correctly drew, at least in theory: if Death and Suffering must be avoided, the only way out of the dilemma is the Buddhist nirvana that is characterised by the absence of all distinctions.

The reason why Schopenhauer cannot rest on Life or Peace without avoiding Suffering or Death is that Life implies Suffering. When Life does not lie within Suffering, as we optimistically have tried to do in the diagram, there is no resting place on Life: only a spike and inhospitable precipices on both side of it (Fig. 22).

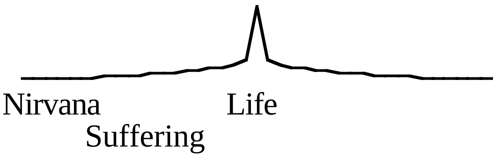

Fig. 22. Suffering and Life disjunct.

Fig. 23. Life inside Suffering.

These conditions are independent of the arrangement of the predicates on the canvass. We can of course take Suffering and drag it to Life, which in this case turns into a vulcano with a landing place on the top where Schopenhauer can stay alive suffering (Fig. 23), but the logic still applies: no Life without Suffering.

3.2. Interpreting the dynamics

The possibility of moving the subject "Schopenhauer" around in the landscape is a useful facility for getting a hands-on experience of the dynamics of the constructed world. It works like a logical cousin to Papert's Turtle world that enables children to explore a mathematical world.

The interpretation of the static features of the system is straightforward. The *locations* of the concepts in the plane represent their actual extensions. For example, if two concepts are distant they have no common members. Some arrangements are more probable than others: for example, it is not probable that Suffering is disjunct from Life as depicted in Fig. 21, since according to common sense Sufferers must be alive. On the other hand, Peace needs not be contained in Death, at least not according to common sense.

The *force-field* of the concept — its *strength, form and domain* — corresponds to the concept's intension, since the task of the fields is to determine which subjects can be combined with which predicates, i.e. determining which sentences are true in this world.

Is it possible to construct an interpretation of the dynamics of the system too? Well, moving the subject "Schopenhauer" around in the plane while observing its behaviour could be taken to represent *analysis* — a thought process: Schopenhauer trying to find a way out of the dilemmas he has created for himself.

But in the model, predicates can be dragged too. The practical purpose is to enable the user to learn how the force-field changes as the extension of the predicates changes. Is there a similar interpretation of "predicate dragging"? In accordance with the rest of the interpretation, "predicate dragging" means *action*, i.e. changing the particular facts of reality, since by dragging a predicate we change the extension of that predicate. We change particular facts, but we do not change the basic regularities of the world represented by the force field created by the logical formulas.

An interesting question now arises: which types of dynamics arise if analysis (moving the subject) and action (moving predicates) occur simultaneously? It turns out that the dynamics and the equilibrium conditions are quite different from the static case.

According to textbooks of philosophy, Schopenhauer did not renounce worldly pleasures (women and food); although he denounced the "lust for life" theoretically, he seems to have a good deal of it himself. This can be represented as an attractor that tries to pull the "Life" predicate towards the "Schopenhauer" subject. What happens if we integrate his philosophy with his own practice in this way? Let him seek Life and Peace!

If we enter vectors that attract Life and Peaceto Schopenhauer and keep Death and Suffering away from him, a pleasant, although rather immoral, dynamics emerges: Life and Peace cling to Schopenhauer, while Death and Suffering keep distance!

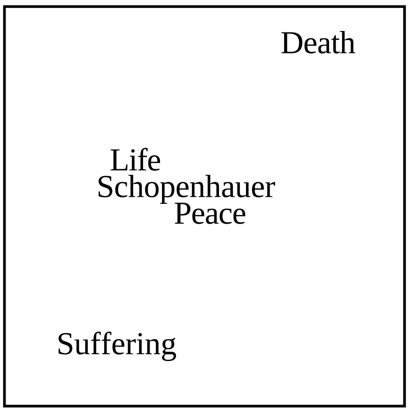

Fig. 24. Lived logic.

Fig. 24 is not stable, because Life and Peace move towards Schopenhauer, whereas Death and Suffering recede. Sometimes Schopenhauer and the two predicates end up moving irregularly around in the middle of the plane, sometimes they rush off to the left or right, but still keep a safe distance to Death and Suffering. If we drag Schopenhauer to Suffering in order to remind him of his philosophy, he quickly leaves it in quest of the middle location.

The continual movements indicate that Schopenhauer never reaches an equilibrium where he is at rest in Life and Peace, so he does not live an honest life — as Bertrand Russell notes in his assessment of Schopenhauer. His life is based on a lie, but still he gets his moments of Pleasure (represented by the system states where the distance between the subject and the predicates are under a certain limit). Whereas *pure* logic defines equilibria in Nirvana or Suffering/Death, the *lived* logic (logic + desire) turns out to occasionally having a cyclic attractor at a very comfortable place: he gets by with a little help from his friends.

The reason for this is probably the following: Life and Peace cling to Schopenhauer and follows him as guardian angles, even if he is forced to suffer; when Peace approaches Suffering together with Schopenhauer, a ridge forms between them, since they are antonyms. Schopenhauer is influenced by this ridge, and moves away from Suffering.

The point of all this is that Schopenhauer's philosophy still reigns in the world we have created. There is no cheating. The only thing we have done is adding new forces of action on top of the Schopenhauer system, but this new combination of analysis and action has completely changed the equilibrium conditions.

Still things are a bit too easy for our philosopher: we have a logic saying that Life implies Suffering, but no forces bring reality in accordance with this law. Life and Suffering has been extensionally disjunct up till now.

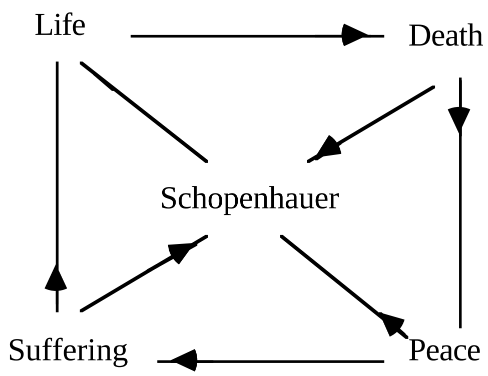

Fig. 25. Lived logic: reality conforming to logic. Legend: A ——▶ B. A repulses B. A ◀—— B A is attracted to B.

What will Schopenhauer do if we add forces that brings reality in closer correspondence with logic? This last experiment is rather complicated. The gradients of the forces are displayed in Fig. 25.

Now reality tries to conform to logic, so that Suffering hunts Life, and Death hunts Peace (the implier attracts the implied, since the former cannot live without the latter, whereas the converse is possible). Death shuns Life, and Suffering Peace. The tormented Schopenhauer hunts Life and Peace, and flees Suffering and Death.

Here are some outcomes of these forces:

1. If Suffering and Death are on the same side of Schopenhauer, he moves forward in disgust, dragging Peace and Life with him, pursued by Death and Suffering which however never catch up with him.
2. If the two dark predicates are on opposite sides, Schopenhauer is trapped, he oscillates on the spot near resounding Death and Suffering. Peace and Life rest immobile on him.

My impression is that even a small system like the present may turn out to be chaotic, since small changes in position and force of the vectors produce unexpected results. Whether it is really chaotic remains to be seen.

3.3. A model of the formal system

In this section I sketch a model for the formal systems defined above. Models are theoretical constructions that provide a semantics for the formal systems, and normally they belong to the set-theoretical type where one-place predicates are mapped into sets, and n-place predicates are interpreted as sets of n-tuples, logical connectors signify operations on these sets, and constants represent members of the sets. The model presented here, however, is not set-theoretic but is borrowed from the theory of autopoiesis (Maturana & Varela, 1980, 1992; Luhmann 1984, 1990).

Life and consciousness are autopoietic systems. They are operationally closed, which means that their environment cannot interfere with their basic reproductive circle: they are autonomous and possess individuality. Luhmann distinguishes between three main system types: *social systems* whose autopoietic circle consists of communications, *psychic systems* whose circle consists of thoughts, and *biological systems* whose closed circle is known as metabolism.

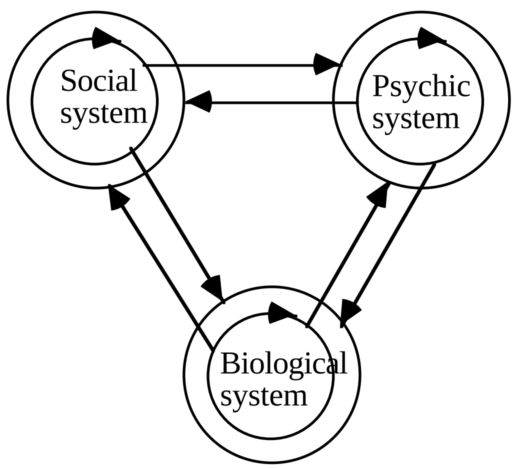

Fig. 26. The model.

The basic distinction runs between a system and its environment. Each system draws that distinction for itself; if several systems exist, then one system sees the others as parts of its environment. Although the environment — with or without other autopoietic systems — cannot interfere with the basic autopoietic circle, it can *perturb* the system. A perturbation is a change of the autopoietic circle — it is thrown into another orbit — but does not concern the domain of autopoietic operations (in social and psychic systems: meaning; in biological systems: life).

Interaction between autopoietic systems is a two-step process:

1. *Perturbation:* the autopoietic circle is thrown out of orbit.
2. *Compensation:* the actual processing of the domain changes because the circular process has changed.

The system discussed in this paper belongs to the psychic systems. The *Consistency Rules (2)* are interpreted as the basic autopoietic recursion whose function is to maintain truth by making subjects seek equilibria if any exists.

Perturbations consist in changes of the force-field. For reasons of space, I have given a very simplistic description in this paper. For example in Section 3.2, where the predicates were made to move according to the "Schopenhauer logic", I very bluntly assumed that predicates move under influence of "reality" (which in this context should be rephrased as "perturbed by its environment"). A fuller description is given in Andersen 1994.

Schopenhauer's own actions in Section 3.2 can be construed as homeostatic echoes of interactions between the psychic and the biological system. His logic is overlaid by actions aiming at acquiring exactly those pleasures which his philosophy teaches are impossible to have. The effect of his actions are fed back into the logical system and is experienced as a new set of forces attracting the positive predicates to the Schopenhauer subject.

The "slow" *Narrative Rules (3)* must also be construed as perturbations, since they do not directly affect the movements of subjects, only the energy landscape in which they move.

Actual narrative rules in real texts presumably are the result of perturbations from all three systems: from the social system in the shape of literary conventions(text linguistics); from the psychic system itself in the form of immanent dispositions for change ("poetic justess, natural endings"), and from the biological system as sexual and aggressive urges. Self-perturbation is not an anomaly but is predicted by the theory; in fact, a system is not able to distinguish a priori between disequilibra induced by itself and by its environment. Such distinctions must learned.

4. Applications of dynamic analysis

In this section I show how dynamic analysis can be applied to two other phenomena, *modality* and *conceptual types.*

4.1. Modality and truth

The stability of the truth of a sentence *Subject + Predicate* is defined by the gradient pulling the Subject if it is placed over the Predicate.

In line with Brandt(1989) we can define *necessity* by the condition that only 1 critical point (a location where the gradient is below ε) exists in the topology. Thus, *Subject + Predicate* is *necessary* if and only if the subject is stable above the predicate and no other critical points for the subject exist. *Subject + Predicate* is *possible*, if the Predicate is a critical point for the Subject and other critical points exist. Thus, the or-operator typically produces possible but not necessary situations (Fig. 5).

Necessity and possibility can develop into each other, as shown in Fig. 6, where two critical points merge into one. Logical inferences can produce the same effect, as shown in Fig. 15-16 where modus tollendo ponens converts the two critical points of $P \vee Q$ into the single one of $(P \vee Q) \wedge \neg P$.

Since we have three systems, the social, the psychic and the biological system, it follows that theoretically we have three variants of modality, one for each system. In the literature two versions are normally recognised, namely epistemic and deontic modality.

Epistemic modality is about signs since it concerns the truth of sentences: “He can be the murderer” = “Maybe he is the murderer, maybe he is not; I don’t know”. Epistemic modality belongs to the social system.

Deontic modality is about abilities and obligations of bodies and minds: “He *can* write well if he chooses to” does not mean that I lack knowledge about his writing abilities; on the contrary, I describe his abilities with complete assurance. Deontic modality belongs to the biological (and possibly the psychic) system.

The epistemic sense of the sentence “He can be the murderer” (but it is also possible that she did it) can be represented as shown in Fig. 27.

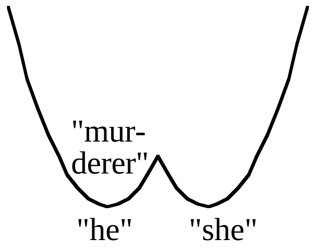

Fig. 27. Epistemic modality

The hypothesis is now that deontic modality has the same definition as the epistemic one, the meaning difference between the two variants being caused by the fact the social, psychic and biological dynamics are different.

Whereas the dynamics of the social system is confined to logical forces of the kind we have described, the dynamics of the biological system consists of physical forces moving body and limbs (Pellionisz & Llinas (1980, 1982) present a neurophysiological account of the perception - action cycle. Arnheim 1974 analyses perception in terms of vectors). Finally, the social systems create attractors or repulsors towards acts (Bourdieu 1993).

Therefore, when we interpret deontic modality, possibility is often interpreted as the ability to overcome some physical *(can, is able to)* or social *(may)* obstacle, whereas necessity connotes the inability to resist one of these forces (Talmy 1988). “He *can* write well” is normally only uttered in a situation where something hinders the action (for example laziness or sloppiness) which is known to be weaker than the opposite force. In opposition to this, “He *must* write” only makes sense in the case where a moral obligation overrules the aforementioned obstacles. In the first case, more than 1 critical points must exist, since we cannot infer “He will write” from “He can write”. In the latter case, however, this inference must be valid, otherwise we were in error in using “must”.

The formal dynamics is common to the three systems; the difference is due to the different substance of the forces: logical, physiological, and normative.

4.2. Predicate types.

Predicates come in different types. A popular distinction is that between Aristotelian and fuzzy concepts.

Aristotelian concepts typically occur in science and are characterised by sharp boundaries: either a tree is a beech or an oak. The concepts are arranged in a Porphyrean tree structured in genus, species and differentiae.

Fuzzy concepts have blurred boundaries: a particular growth can lie somewhere between a tree and a shrubbery. Fuzzy concepts are frequent in everyday language: “Is she your girlfriend? Well, yes and no....”.

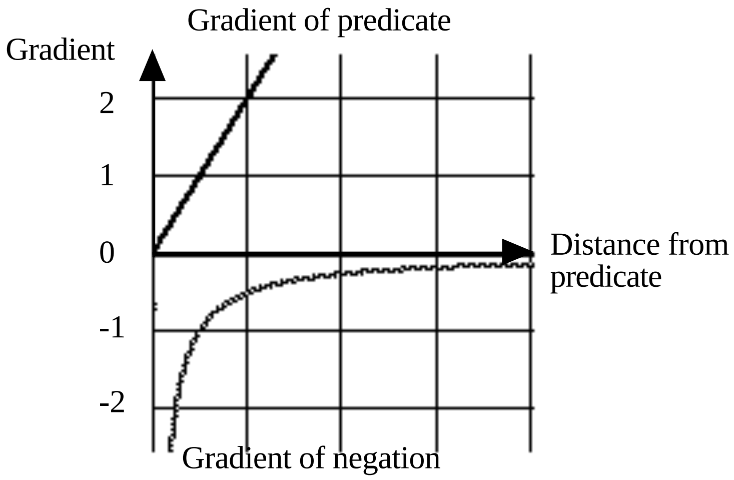

Fig. 28. The gradients of predicate and its negation as a function of the distance from predicate

This distinction is predicted by the notion of conceptual inertia. In our construction, we assumed that subjects possess inertia and that a force larger than some threshold ε is necessary to move the subject at all. If we assume that the force-field of a concept is given by x^2, then Fig. 28 displays the gradients working on a subject; the black curve is the positive attracting force of the predicate, whereas the grey curve is the negative repulsing force of its negation.

Inspection of Fig. 28 shows that both the concept and its negation achieve an absolute force of 1 when the distance is 0.5[10]. This means that the gradient of the predicate exceeds 1 to the right of 0.5 and that of the negation does so to the left of 0.5. This again entails that there is an area around 0.5 where neither predicate nor negation can move a heavy subject whose ε-limit is larger than 1. The force is simply too small. If a heavy subject lies in this grey zone[11], it will stay unaffected by predicate and its negation, a behaviour characteristic of *fuzzy concepts.* One can truthfully say “This is a bush” and “This is not a bush”.

Now if ε = 1 there is only one point where negation-processes do not affect the subject, namely 0.5. To the left of this point, negation will expel the subject, and to the right the predicate will attract it. There is no “both-and” zone. This behaviour is typical of *Aristotelian concepts.*

Interestingly enough the model predicts a third type of concepts, namely light *litotes* with thresholds below 1. If ε < 1 there is an area where neither predicate nor negation are stable. For example, for ε = 1/2, the inequality |2x|

$\leq \frac{1}{2}$, $\left|\frac{-1}{2x}\right| \leq \frac{1}{2}$ has no solutions. This means that the subject slides "too far" when we negate a predicate: it finds a resting place beyond the area in which it was stable in P. This conceptual type is very common in daily speech: "He is certainly not poor" often means "He is rich". The subject not only slides out of "poor", its momentum carries it well into the area of "rich".

5. Summary

In this paper I have described a logic system as a dynamic hyperspace. Besides two kinds of objects, subjects and predicates, the space contains force fields that influence subjects. The field is anchored in the predicates, but is modulated by logical formulas that specify the basic laws of the space. The field can be perturbed, either by internal or external forces. Perturbations change its equilibrium conditions, which sets the subjects in motion, which again changes the truth-values of sentences.

The basic logical process is a perpetual cycle of slow perturbations that upset the force field and fast compensations that maintain truth in the space.

The construction is practically motivated by design of interactive media, which require us to construe the product as a set of possibilities instead of seeing it as a fixed work of art — as a dance in the sense of a set of regular patterns, and not as a particular performance.

Apart from this practical motivation, I have argued that the field concept can better cope with dilemmas and contradictions, and predicts phenomena that in fact can be found in human language and behaviour. Examples of the latter are: conceptual types (e.g. Aristotelian versus fuzzy concepts), effects of negation (e.g. natural candidates for replacing the negated predicate), and classes of truth-value behaviour: stable and unstable, cyclic and non-cyclic types.

References

ANDERSEN, P. BØGH & B. HOLMQVIST (1990). Interactive fiction. Artificial intelligence as a mode of sign production. *AI and Society 4* : 291-313.

ANDERSEN, P. BØGH (1992). Vector spaces as the basic component of interactive systems. Towards a computer semiotics. *Hypermedia* 4(1): 53-76.

ANDERSEN, P. BØGH (1994). Katastrophen und computer. *Zeitschrift für Semiotik* 16(1/2): 29-50.

ANDERSEN, P. BØGH & P. ØHRSTRØM (1994). Hyperzeit. *Zeitschrift für Semiotik* 16(1/2): 51-68.

ANDERSEN, P. BØGH (1995). The force dynamics of interactive system. Towards a computer semiotics. *Semiotica* 103(1/2).

ANDERSEN, P. BØGH. The semiotics of autopoiesis. A catastrophe theoretic approach. *Cybernetics & human knowing* 2(4): 17-38.

ARNHEIM, R. (1974). *Art and visual perception.* Univ. of California Press.

BOURDIEU, P. (1993). *The Field of Cultural Production.* Cambridge: Polity Press.

BRANDT, P. AA. (1989). Agonistique et analyse dynamique catastrophiste du modal et de l'aspectuel. *Semiotica 77(1/3):* 151-162.

GÄRDENFORS, P. 1992. Psykiske forestillinger, begrebslige rum og metaforer [Psychic images, conceptual spaces and metaphors]. In: *Litteratur og samfund* 47/48: 54-81.

GREIMAS, A. J. & J. COURTÉS (1979). *Semiotique: Dictionnaire raisonné de la théorie du language.* Paris: Hachette.

GUPTA, A. & N. BELNAP (1993). *The Revision Theory of Truth.* Cambr., Mass: The MIT Press.

HERZBERGER, H. G. (1982) Notes on naive semantics. *Journal of Philosophical Logic* 11: 61-102.

KRIPPENDORFF, K. (1994). A Recursive Theory of Communication. In: David Crowley & David Mitchell (eds): *Communication Theory Today,* 78-104. Cambridge: Polity Press.

LOTMAN, Y. M. (1990). *Universe of the mind.* Bloomington and Indianapolis: Indiana University Press.

LUHMANN, N. (1984). *Soziale Systeme.* Frankfurt am Main: Suhrkamp.

LUHMANN, N. (1990). *Essays on Self-Reference.* New York: Columbia University Press.

MATURANA, H. R. & F. J. VARELA (1980). *Autopoiesis and Cognition.* The Realization of the Living. D. Reidel Publ. Comp.: Dordrecht/Boston/London.

MATURANA, H. R. & F. J. VARELA (1992). *The tree of knowledge.* Shambala: Boston & London.

MONTAGUE, R. (1976). *Formal Philosophy.* Yale University Press: New Haven and London.

PELLIONISZ, A. & R. LLINAS (1980). Tensorial approach to the geometry of brain function: cerelallar coordination via a metric tensor. *Neuroscience 5:* 1125-1136.

PELLIONISZ, A. & R. LLINAS (1982). Space-time representation in the brain. The cerebellum as a predictive space-time metric tensor. *Neuroscience 7:* 2949-2970.

PETITOT, J. & R. THOM (1983). *Sémiotique et théorie des catastrophes. Actes Sémiotiques 5(47/48).* Paris: Institut National de la Langue Francaise.

PETITOT, J. (1985). *Morphogenese du sens I.* Paris: Presses Universitaires de France

PETITOT, J. (1989). On the linguistic import of catastrophe theory. *Semiotica 74(3/4):* 179-209.

SAUNDERS, P. T. (1990). *An introduction to catstrophy theory.* Cambridge: Cambridge University Press.

TALMY, L. (1988). Force dynamics in language and cognition. *Cognitive Science 12:* 49-100.

THOM, R. (1983). *Mathematical models of morphegenesis.* Chichester: Ellis Horwood.

THOM, R. (1990). *Semio Physics. A sketch.* Redwood City, CA: Addison-Wesley Publ. Comp.

THOM, T. (1989). *Structural Stability and Morphogenesis.* Redwood City, Calif: Addison-Wesley.

WILDGEN, W. (1982). *Catastrophe Theoretic Semantics.* Amsterdam: John Benjamins Publ. Comp.

WILDGEN, W. (1985). *Archentypen-semantik.* Tübingen: Gunter Narr Verlag.

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[1] This is essentially the method suggested in Gupta, & Belnap (1993)

[2] This argument can be traced formally as follows. The problematic sentence, "All members of capitalist society have a false consciousness" is called (a) and is formalized as follows:

(a) $\forall x: x \in CSociety \rightarrow FalseConsciousness(x)$.

Then make the following deductions:

(1) $\forall x: x \in CSociety \rightarrow FalseConsciousness(x)$. Our axiom.

(2) $\forall x, y: FalseConsciousness(x) \wedge Assert(x, y) \rightarrow \neg y$. What a person with false consciousness utters is false. Definition.

(3) $I \in CSociety$. I, the author, is a member of a capitalist society. Factual truth, but necessary in all cases where social systems are criticized from within.

(4) $FalseConsciousness(I)$. Therefore, by (1) and (3) I suffer from false consciousness.

(5) $Assert(I, a)$. I assert *a*, namely that all members of capitalist society have a false consciousness. Follows from the speech-act itself.

(6) $\neg a$. Therefore, by 2, 4 and 5, a must be false. Substituting the proposition for its name we learn that

(7) $\neg\forall x: x \in CSociety \rightarrow FalseConsciousness(x)$ which by $\neg\forall p \equiv \exists p\neg$ yields

(8) $\exists x: x \in CSociety \wedge \neg FalseConsciousness(x)$. There is at least one member of a capitalist society that does not have false consciousness.

[3] The formal deduction could go like this (M = it is possible, N = it is necessary).

(1) $\forall p, M\neg p$. Axiom: All sentences may be false, there are no necessary sentences.

(2) $M\neg(\forall p, M\neg p)$. Since (1) is a sentence, (1) is possibly wrong. Therefore we can substitute (1) for p in (1).

(3) $M(\exists p, \neg M\neg p)$. By $\neg\forall p \equiv \exists p\neg$.

(4) $M(\exists p, Np)$. Using $\neg M\neg \equiv N$ gives us that possibly a necessary sentence exist.

[4] I am grateful to Svend Østergård for help with the mathematics.

[5] f(P $\vee$ Q) = min(f(P),f(Q)). In the following, the potential associated to the predicates are all rendered by the formula $y = x^2$. The real force field of the predicates is an empirical matter.

[6] Note that the resulting potential is not differentiable: although the peak between P and Q looks like a maximum, it is not, since the function is not differentiable at that point. This is a theoretical problem, since we use the first derivative as the force controlling the logic, and if the potential is not differentiable for all values, the space will have unmotivated "holes" in it where the gradient is undefined.

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[7] f(P ∧ Q) = max(f(P),f(Q)).

[8] One possibility would be that $g' = -\frac{1}{h'}$ where h' is the derivative of f(P), which in our case is 2x, and g' is the derivative of f(¬P). Thus, the new derivative should be the negative and reciprocal of the old derivative. This gives us the following formula for negation: $f(\neg P) = -\int \frac{1}{f'(P)}$. As we shall see, this definition of negation describes our linguistic intuitions in a number of cases, but it is complicated to calculate. Why not choose simple negation, so that f(¬P) = -(f(P))? Simple negation will give us a truly Boolean logic where e.g. de Morgan's law obtains: f(¬(P ∨ Q)) = -f(P ∨ Q) = -min(f(P), f(Q)) = max(-f(P), -f(Q)) = f(¬P ∧ ¬Q).

The problem is that this negation is very difficult to interpret. For example, x^2 and $-x^2$ will have the same critical point, so negating a predicate will not cause a subject whose gradient is 0 to move. Going from "The earth is flat" to "The earth is not flat" will not change our beliefs, in glaring opposition to the facts. Also, the negation will move the predicate slowly inside P, and will increase the speed as we remove ourselves from P. We would expect the opposite: if we negate a predicate P, then subjects of which P is true must change position, whereas subjects outside the stability of P can be more at ease. These are the reasons for choosing the more complicated version.

In order to find the function g in the case where f(P) = x^2, we note that

$$g = \int g' = \int -\frac{1}{h'} = -\int \frac{1}{h'} = -\int \frac{1}{2x} = \frac{-\ln|x|}{2}$$ (plus some constant C).

which suggests that negation of curves of the form x^2 should be represented by the negative of the natural logarithm of the original equation divided by 2.

Observe also that two negations cancel each other as they should: Neg(Neg(h')) = Neg($-(h')^{-1}$) = $-(-(h')^{-1})^{-1}$ = h'.

[9]The size of the problem depends on the choice of ε. The two curves of P and ¬P intersect approximately in ±0.55. If ε = 1 then the gradient of x^2 counts as 0 in the interval -0.5... +0.5 and that of -(ln|x|)/2 counts as 0 in the intervals 0.5...∞, and -∞ ...-05. Therefore, we have only problems in the intervals -0.55...-0.50 and 0.50...0.55, where the x^2 furnishes the segment of the curve but has a too large gradient. If ε = 1.5 there is no problem, since the gradient counts as zero everywhere.

[10] The gradient of the concept is |2x|, and that of the negation is $\left|\frac{-1}{2x}\right|$. The inequalities |2x| ≤ 1 and $\left|\frac{-1}{2x}\right| \le 1$ has the unique solution x = ±1/2.

[11] For ε = 2 the inequality |2x| ≤ 2, $\left|\frac{-1}{2x}\right| \le 2$ has solutions in the interval -1...-1/4 and l/4...+1. In this interval, subjects will remain stable when we negate a concept.